Class:XI	Section:	Name:

LAHORE GRAMMAR SCHOOL
55 MAIN GULBERG, LAHORE

Additional Mathematics **4037/1**
PAPER 1

2 hours

Candidates answer on the Question Paper.
Additional Materials: Geometrical Instruments **Electronic calculator**

READ THESE INSTRUCTIONS FIRST

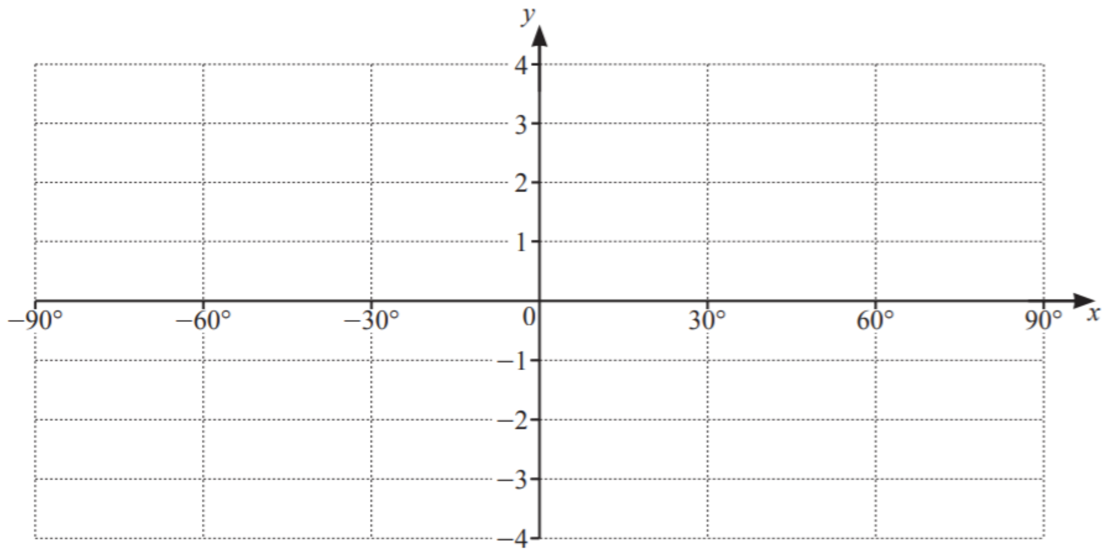
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the question paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, papers clips, highlighters, glue or correction fluid.

Answer all questions.
The number of marks is given in brackets [ ] at the end of the each question or part question.

If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is **80.**

This document consists of 14 printed pages. [turn over]

1 **(i)** On the axes below, sketch the graph of $y = 2\cos 3x - 1$ for $-90° \leqslant x \leqslant 90°$.



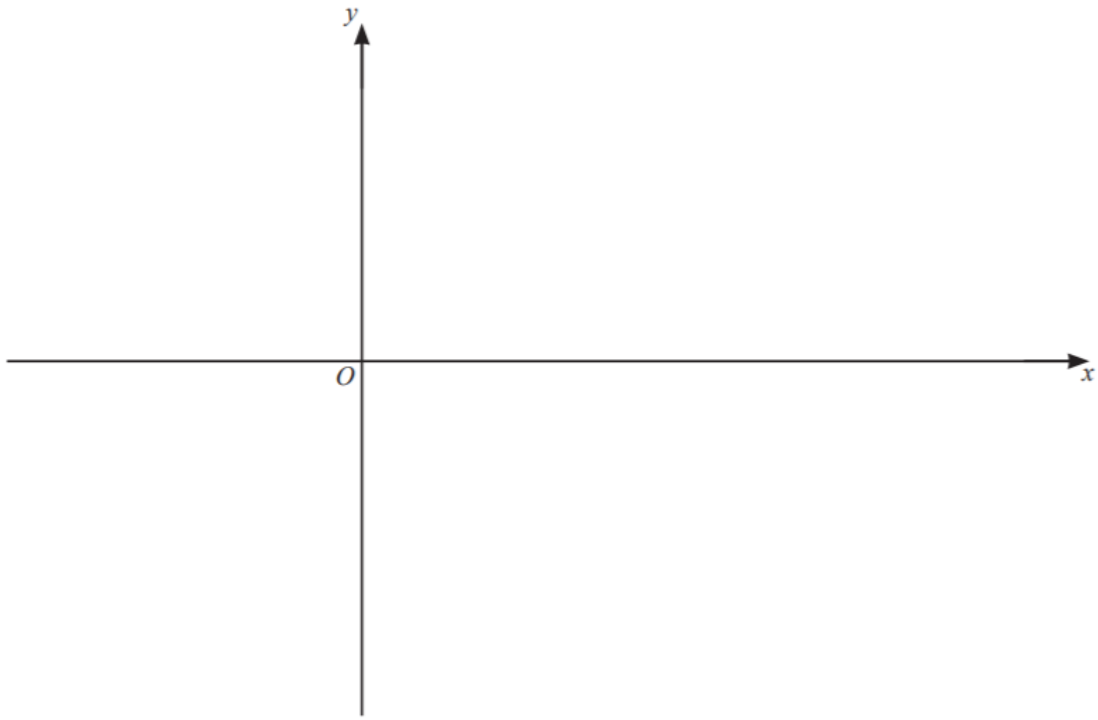
[3]

(ii) Write down the amplitude of $2\cos 3x - 1$. [1]

2 When $\lg y^2$ is plotted against x, a straight line is obtained passing through the points $(5, 12)$ and $(3, 20)$. Find y in terms of x, giving your answer in the form $y = 10^{ax+b}$, where a and b are integers. [5]

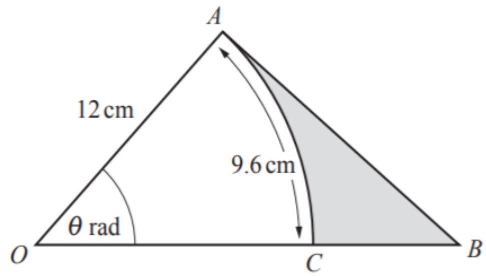
3 The first three terms in the expansion of $\left(1-\frac{x}{7}\right)^{14}(1-2x)^{4}$ can be written as $1+ax+bx^{2}$. Find the value of each of the constants a and b. [6]

4 **(i)** On the axes below, sketch the graph of $y = |2x^2 - 9x - 5|$ showing the coordinates of the points where the graph meets the axes. [4]



(ii) Find the values of k for which $|2x^2 - 9x - 5| = k$ has exactly 2 solutions. [3]

5



The diagram shows the right-angled triangle OAB. The point C lies on the line OB. Angle $OAB = \frac{\pi}{2}$ radians and angle $AOB = \theta$ radians. AC is an arc of the circle, centre O, radius 12 cm and AC has length 9.6 cm.

(i) Find the value of θ. [2]

(ii) Find the area of the shaded region. [4]

6 **(a)** Eight books are to be arranged on a shelf. There are 4 mathematics books, 3 geography books and 1 French book.

(i) Find the number of different arrangements of the books if there are no restrictions. [1]

(ii) Find the number of different arrangements if the mathematics books have to be kept together. [3]

7 It is given that $y = (1+e^{x^2})(x+5)$.

(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the approximate change in y as x increases from 0.5 to $0.5+p$, where p is small. [2]

(iii) Given that y is increasing at a rate of 2 units per second when $x = 0.5$, find the corresponding rate of change in x. [2]

The polynomial $p(x) = ax^3 + bx^2 + cx - 9$ is divisible by $x + 3$. It is given that $p'(0) = 36$ and $p''(0) = 86$.

(i) Find the value of each of the constants a, b and c. [6]

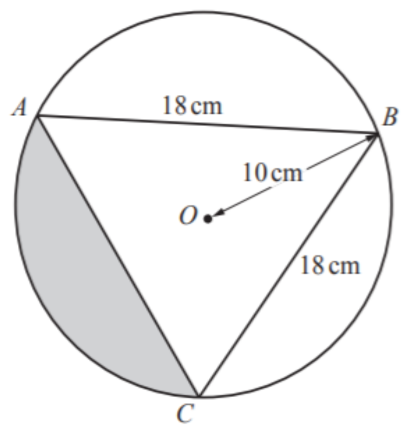
(ii) Using your values of a, b and c, find the remainder when $p(x)$ is divided by $2x - 1$. [2]

9 A solid circular cylinder has a base radius of r cm and a height of h cm. The cylinder has a volume of $1200\pi \text{ cm}^3$ and a total surface area of $S \text{ cm}^2$.

(i) Show that $S = 2\pi r^2 + \frac{2400\pi}{r}$. [3]

14

10



The diagram shows a circle centre O, radius 10 cm. The points A, B and C lie on the circumference of the circle such that $AB = BC = 18$ cm.

(i) Show that angle $AOB = 2.24$ radians correct to 2 decimal places. [3]

(ii) Find the perimeter of the shaded region. [5]

(iii) Find the area of the shaded region. [4]

16

11 A curve is such that $\frac{d^2y}{dx^2} = 2(3x-1)^{-\frac{2}{3}}$. Given that the curve has a gradient of 6 at the point (3, 11), find the equation of the curve. [8]

11 Answer only **one** of the following two alternatives.

EITHER

A curve has the equation $y = xe^{2x}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]

(ii) Show that the y-coordinate of the stationary point of the curve is $-\frac{1}{2e}$. [3]

(iii) Determine the nature of this stationary point. [2]

OR

(i) Show that $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = \frac{1 - 2\ln x}{x^3}$. [3]

(ii) Show that the y-coordinate of the stationary point of the curve $y = \frac{\ln x}{x^2}$ is $\frac{1}{2e}$. [3]

(iii) Use the result from part **(i)** to find $\int\left(\frac{\ln x}{x^3}\right)dx$. [4]

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

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[Turn over]

1 Given that $y = \frac{\sin x}{\ln x^2}$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [4]

2 Given that $y = 2\sin 3x + \cos 3x$, show that $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = k\sin 3x$, where k is a constant to be determined. [5]

3 A curve has equation $y = \dfrac{x^3}{\sin 2x}$. Find

(i) $\dfrac{dy}{dx}$, [3]

(ii) the equation of the tangent to the curve at the point where $x = \dfrac{\pi}{4}$. [3]

4 Solve

(i) $2^{3x-1} = 6,$ [3]

(ii) $\log_3(y+14) = 1 + \dfrac{2}{\log_y 3}$. [5]

5 **(i)** Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs. [2]

6 **(i)** Show that $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}=\dfrac{2}{\sin x}$. [5]

(ii) Hence solve the equation $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}=1+3\sin x$ for $0^\circ \leqslant x \leqslant 180^\circ$. [4]

Find the coordinates of the points where the line $2y - 3x = 6$ intersects the curve $\frac{x^2}{4} + \frac{y^2}{9} = 5$. [5]

8 **(i)** Show that $\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 2 \tan x \sec x$. [4]

(ii) Hence solve the equation $\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = \operatorname{cosec} x$ for $0° \leqslant x \leqslant 360°$. [4]

(a) Find $\int \sqrt[3]{2x-1}\,dx$. [2]

(b) **(i)** Find $\int \sin 4x\,dx$. [2]

(ii) Hence evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x\,dx$. [2]

10 Two lines are tangents to the curve $y = 12 - 4x - x^2$. The equation of each tangent is of the form $y = 2k + 1 - kx$, where k is a constant.

(i) Find the two possible values of k. [5]

(ii) Find the coordinates of the point of intersection of the two tangents. [4]

11 The functions f and g are defined for real values of $x \geqslant 1$ by

$$f(x) = 4x - 3,$$

$$g(x) = \frac{2x+1}{3x-1}.$$

(i) Find $gf(x)$. [2]

(ii) Find $g^{-1}(x)$. [3]

The gradient of the normal to a curve at the point with coordinates (x, y) is given by $\frac{\sqrt{x}}{1-3x}$.

(i) Find the equation of the curve, given that the curve passes through the point $(1, -10)$. [5]

(ii) Find, in the form $y = mx + c$, the equation of the tangent to the curve at the point where $x = 4$. [4]

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